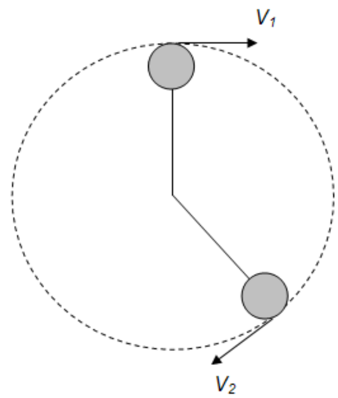
Uniform Circular Motion

Uniform Circular Motion (UCM)
An object that moves in a circle at constant speed 'v'. The magnitude of the velocity remains constant, but the direction of the velocity is continuously changing as the object moves around the circle.

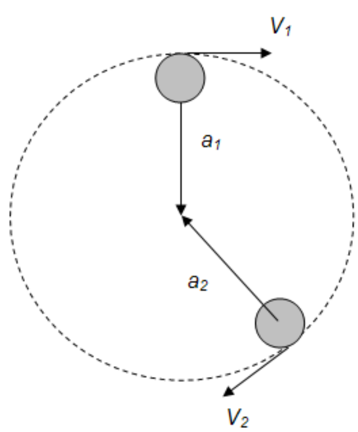
Note that at each point, the instantaneous velocity is in a direction tangent to the circular path.

Since acceleration is the rate of change of velocity, a change in direction of velocity constitutes an acceleration just as does a change in magnitude.



An object revolving in a circle is continuously accelerating even though the speed v, remains constant.

Centripetal Acceleration
Whenever an object is undergoing UCM the acceleration is directed towards the center of the circle – this is called centripetal acceleration. The magnitude of the centripetal acceleration is constant, but the direction changes each instant, so that the acceleration is always directed toward the center of the circle.



Formula:

$$a_c = \frac{v^2}{r}$$

Where

a_c = centripetal acceleration (m/s^2)

v = velocity (m/s)

r = radius of the circular path (m)

Summary:

- **Acceleration vector points toward the centre of the circle**
- **Velocity vector points in the direction of motion which is tangential to the circle**
- **Velocity and acceleration vectors are perpendicular to each other at every point in the path for UCM**

UCM can also be described in terms of the frequency (number of revolutions per second) and period (time required for one complete revolution).

Recall:

$T = \frac{1}{f}$ and $f = \frac{1}{T}$

Since the distance travelled in one revolution is the circumference of the circle ($C = 2\pi r$) and the time is the period of rotation, we can write:

$$v = \frac{d}{t}$$

$$v = \frac{2\pi r}{T}$$

for an object revolving in a circle at a constant speed.

Example:

A 150.0 g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

Example:
The outer edge of a 120.0 mm diameter DVD experiences an acceleration of 1.6 m/s^2. What is the speed of the DVD?

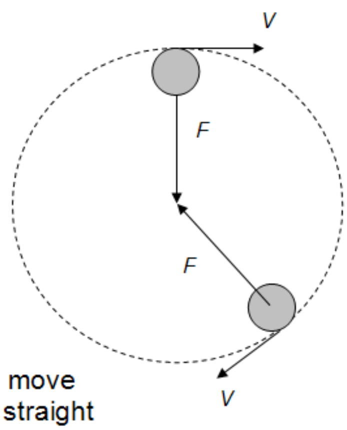
Example:
The moon's nearly circular orbit about Earth has a radius of about 384 000 km and a period of 27.3 days. Determine the acceleration of the moon toward Earth.

Centripetal Force and Newton's Laws

Recall Newton's second law: $F_{net} = ma$.

An object that is accelerating must have a net force acting on it. This net force is in the direction of the acceleration (toward the centre of the circle for UCM).

A force is required to keep an object moving in a circle. If the speed is constant the force is directed toward the centre of the circle.



If the net force were to become 0, the object would move off at a constant speed in a straight line. To prevent this a net force toward the centre of the circle is necessary.

Since,

$$F_{centripetal} = F_{net} = ma_c$$

then

$$F_c = \frac{mv^2}{r}$$

The direction of the net force is continually changing so that it is always directed to the centre of the circle. This force is called a centripetal force.

*Note that centripetal force is not a new force – it is simply the direction of the net force in UCM. The force must be applied by other objects (eg. a person's hand swinging a ball around).

You may have heard of centrifugal force as the force acting outward on an object in UCM. There is no such force. This misconception probably stems from the feeling you get when spinning a ball on a string, of your hand being pulled outward. To keep the ball moving in a circle, you actually pull inward on the string which in turn exerts a force on the ball. The ball exerts an equal and opposite force (Newton's third law) which is what you feel. If centrifugal force did exist, a ball spun on a string should outward if the string breaks, but this is not what happens.

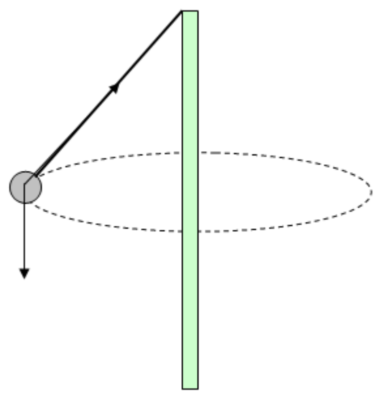
Centripetal Force in a Horizontal Plane

Example:
A child on a merry-go-round is moving with a speed of 1.35 m/s when 1.20 m from the centre of the merry-go-round.
Calculate:
a) the centripetal acceleration of the child
b) the net horizontal force exerted on the child.

Example:
Estimate the force a person must exert on a string attached to a 0.150 kg ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second.

*Note that the ball's weight complicates matters, making it impossible to revolve a ball with the cord perfectly horizontal. But if the weight is small enough we can ignore it and the tension in the string will act nearly horizontally and provide the force necessary to give the ball its centripetal acceleration.

Example:
The game of tetherball is played with a ball tied to a pole with a string. When the ball is struck it whirls around the pole as shown. In what direction is the acceleration of the ball, and what causes the acceleration?



The acceleration points horizontally toward the centre of the ball's circular path. The vertical component of the string tension balances the ball's weight mg. The horizontal component of the string tension, F_{TX}, is the force that produces the centripetal acceleration toward the centre.

Centripetal Acceleration in a Vertical Plane

Example:
A 0.335 kg ball is twirled vertically on the end of a 96.5 cm rope at a constant speed of 3.15 m/s. What is the tension in the rope at the top of the swing?

Example:
A 0.150 kg ball on the end of a 1.10 m long cord of negligible mass is swung in a vertical circle.

a) Determine the minimum speed the ball must have at the top of its arc so that it continues moving in a circle.
b) Calculate the tension in the cord at the bottom of the arc, assuming that the ball is moving at twice the speed in part a).

Example:
A 55 kg rider on a ferris wheel moves in a vertical circle of radius 9.0 m at a constant speed of 3.0 m/s. What is the normal force that the seat exerts on the rider at the top and at the bottom of the circle?

Example:
In an automatic clothes dryer a hollow drum moves the clothes in a vertical circle of diameter 0.75 m. The dryer is designed so that the clothes tumble and do not simply stick to the drum as it rotates. Calculate the speed at which the drum must rotate so that a 0.425 kg sweater at the top of the drum will just begin to tumble.

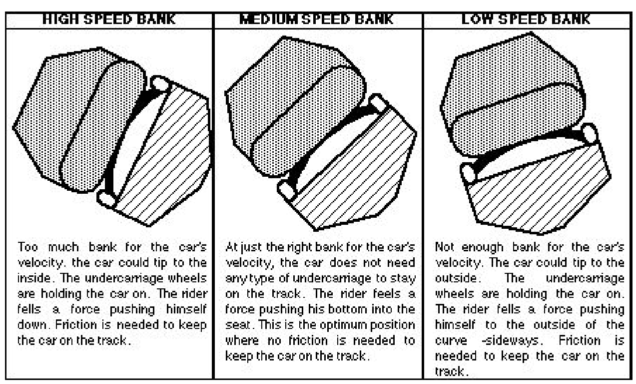
Centripetal Force and Banked Curves

An automobile rounding a curve is an example of centripetal acceleration. To continue moving on a curved path, a car must have an inward force exerted on it. On a flat road this inward force is supplied by friction between the tires and the pavement. This is static friction as long as the tires are not slipping. If the frictional force is too small as in icy conditions, the car will skid out of its circular path.

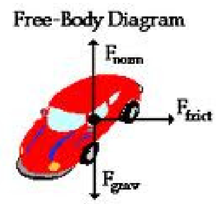
The banking of curves can reduce the chance of skidding because the normal force of the road will have a component toward the centre of the circle, thus reducing the reliance on friction. For a given banking angle there will be one speed for which no friction at all is required. This will occur when,

$$F_N \sin\theta = \frac{mv^2}{r}$$

The banking angle of a road is chosen so that this condition holds for a particular speed called the “design speed”.

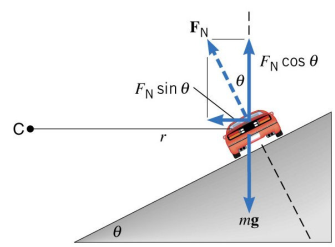


i) forces on a car rounding a curve on a flat road.



ii) forces on a car rounding a banked curve.

Derivation:



Example:
What is the banked angle for an expressway off-ramp curve of radius 50.0 m at a design speed of 50.0 km/h?

Example:
A racecar travels along a banked curve at a speed of 120 km/h. It does not depend on the force of friction to keep it on the track. If the turn is banked at an angle of 25^o to the horizontal, what is its radius of rotation?

Example:
A 1500 kg car travels at 25 m/s around a circular curve on a flat road. If the coefficient of static friction is 0.750, calculate the minimum radius of curvature the car can make.