Lesson 4

Elastic Potential Energy and Hooke's Law

A stretched spring can also possess stored energy. This is called elastic potential energy (E_e).

Since a stretched spring can do work as it returns to its original position, it must possess stored energy. The same is true for an elastic band or an arrow pulled back in a bow.

Springs can have different amounts of stiffness, as indicated by the spring constant k.

Activity: Spring constant.

Hooke's Law:
The restoring force of a spring is given by,

$$F = kx$$

Where,
F = applied force (N)
k = spring constant (N/m)
x = amount stretched from the rest position (m)

Example:
The spring constant for a tire gauge is 3.0×10^2 N/m. When the tire gauge is pushed onto the valve stem of the tire the bar indicator extends 1.9 cm. What force does the air in the tire apply to the spring?

Example:
What is the spring constant for a mass of 2.0 kg hanging on a spring stretched 4.0 cm from its rest position?

Note: When a spring is stretched, we generally use positive values for F and x; we use negative values when a spring is compressed (squeezed).

If too much force is applied to a spring, it may become permanently deformed or may even break. Use of excessive force destroys the elasticity of the spring.

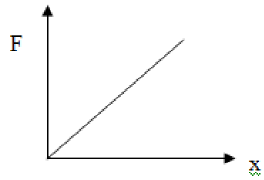
Energy in a Spring

The energy stored in a spring (elastic potential energy) has the ability to do work.

$$E_e = \frac{kx^2}{2}$$

Where,
E_e = elastic potential energy (J)
k = spring constant (N/m)
x = amount of stretch or compression (m)

We can look at the work required as the area under a graph of force vs. stretch.



$$Area = \frac{1}{2}bh$$

$$Area = \frac{1}{2}Fx$$

$$but\ F = kx$$

$$So,$$

$$Area = \frac{1}{2}kx \cdot x$$

$$Area = \frac{kx^2}{2} = work$$

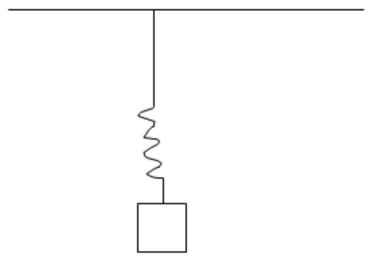
Example:
A spring with a force constant of 240 N/m has a 0.80 kg mass suspended from it. What is the extension of the spring and how much elastic potential energy does it have once the mass is suspended?

Example:
How much work must be done to compress a spring 4.0 cm if the spring constant is 55 N/m?

Example:
A toy gun has its spring compressed 3.0 cm by a 50.0 g projectile. The spring constant is 4.0×10^2 N/m. Calculate the velocity of the projectile if it is launched horizontally.

Simple Harmonic Motion

Consider a mass hanging from a spring as shown.



The mass has an equilibrium position where it is not moving. If the spring is stretched the spring will oscillate back and forth past the equilibrium point until equilibrium is achieved. Ideally the mass should move an equal distance above and below the equilibrium point. This does not actually happen due to damping and friction.

Motion that obeys Hooke's Law is called simple harmonic motion (SHM). The total energy of a mass-spring system is given by:

$$E_T = \frac{mv^2}{2} + \frac{kx^2}{2}$$

*Note that at maximum compression, the kinetic energy is 0 because the system is motionless. As the mass passes the equilibrium point, there is no potential energy since x=0.

Example:
Given a mass-spring system with a bob of mass 0.485 kg, a spring constant of 33 N/m and an initial displacement of 0.23 m determine,

a) the kinetic energy of the bob as it passes the equilibrium point.
b) the bob's speed as it passes the equilibrium point?

To calculate the acceleration of a mass on a spring, note that ,

$$F_{net} = F_{spring}$$
$$ma = kx$$
$$a = \frac{kx}{m}$$

Example:
A 2.0 kg mass on a spring is extended 0.30 m from the equilibrium position and released. The spring constant is 65 N/m.

a) What is the initial elastic potential energy of the spring?
b) What maximum speed does the mass reach?
c) At 0.20 m displacement, what is the speed of the mass?
d) What is the maximum acceleration?
e) What is the acceleration when the displacement is 0.20 m?

Example:
A spring stretches 0.150 m when a 0.300 kg mass is hung from it. The spring is then stretched an additional 0.100 m from this equilibrium point an released. Determine:

a) the spring constant
b) the maximum velocity.
c) the velocity when the mass is 0.050 m from equilibrium.
d) the maximum acceleration of the mass

The period of simple harmonic motion (time for one complete cycle) is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$